We see the modern psychologist rather in the laboratory than in the study room[1] – The alley experiments by Franz Hillebrand (1863-1926)

Pierre Sachse*, Ursula Beermann**, Peter Goller***, Stefan E. Huber*/****,

Marco R. Furtner*****, Thomas Maran******, Robert Marhenke*, Hisaaki Tabuchi*,

Alexandra Hoffmann*, Christian Büsel* & Markus Martini*

* Leopold-Franzens-University of Innsbruck, Department of Psychology
** UMIT – Private University for Health Sciences, Medical Informatics and Technology, Hall i.T.
*** Leopold-Franzens-University of Innsbruck, Archives
**** Leopold-Franzens-University of Innsbruck, Department of Basic Sciences in Engineering Sciences
***** University of Liechtenstein, Institute of Entrepreneurship, Vaduz
****** Leadershipwerk, Vaduz, Liechtenstein

ABSTRACT

The experimental psychologist Franz Hillebrand (1863-1926), who had been trained by Ewald Hering and Ernst Mach, worked in Innsbruck for three decades. His scientific research during this time focused on experimental investigations of spatial perception. His pioneering „alley experiments" initiated the clarification of the important question of the geometrical structure of visual space. His study results suggest that visual space is inhomogeneous with respect to its geometry, which he assumes to be locally Euclidean or hyperbolic. Initiated by Hillebrand's experiments, research received crucial and internationally visible input in a variety of fields such as spatial perception, the geometry of visual-spatial orientation, size constancy, as well as in the development of mathematical models and theories such as in the area of perception psychology. Additionally to the history and the results of the above-mentioned studies, the life and work of the pioneer of this research, Franz Hillebrand, will be introduced.

Keywords

Visual space perception – geometry of visual space – non-euclidean space perception – alley experiments – Franz Hillebrand

1 Introduction

„You have shown a way to eradicate an old psychological superstition, for which I offer you my sincere congratulations", the experimental physicist Ernst Mach (1838-1916) wrote to the experimental psychologist Franz Hillebrand on June 21st, 1901, in Innsbruck (UAI Nachlass Franz Hillebrand). Mach addressed hereby the extraordinary achievement of Hillebrand's text „Theory of the apparent size in binocular vision" (orig. „Theorie der scheinbaren Grösse bei binocularem Sehen"), submitted in 1901 at the Vienna Academy and published there in 1902. With this publication, Hillebrand was the first to analyze the problem of non-Euclidean (hyperbolic) spatial perception by means of the „alley experiments". With these studies he initiated the clarification of the significant question of the geometrical structure of the visual space. In a presentation held in 1870 in the Lecturer Society of Heidelberg, the physiologist and physicist Hermann Helmholtz (1821-1894) had already pointed out that our perception by no means was restricted on Euclidean relations in the physical world (cf. von Helmholtz, 1884).

[1] UAI Nachlass Franz Hillebrand, inaugural lecture on October 19, 1896, „Experimental Psychology, its formation and its tasks" (orig. „Die experimentelle Psychologie, ihre Entstehung und ihre Aufgaben").

Hillebrand's tractate „The relation of accommodation and convergence on depth localization" (orig. „Das Verhältnis von Accomodation und Konvergenz zur Tiefenlokalisation") (1894) can be seen as preliminary work for the „alley experiments". Also the experiments of Götz Martius (1889), Johannes von Kries (1891), and Wilhelm Holtz (1893), with which Hillebrand was very familiar, might have had an effect on his „alley experiments". The crucial initiator and precursor for Hillebrand's experiments, however, was without doubt the physiologist Ewald Hering (1843-1918), who, moreover, was one of his most formative teachers in Prague. Hering, with his research on physiology (of the senses), is part of the „age of classical psychophysics", of researchers such as Johannes Müller, Jan Evangelista Purkinje, Ernst Heinrich Weber, Alfred Wilhelm Volkmann and Gustav Theodor Fechner.

Hillebrand's study participants in Innsbruck – among them his faculty colleague Wilhelm Wirtinger (Mathematics), Paul Czermak (Experimental Physics), and Michael Radakovic (Theoretical Physics), as well as his wife Silvia Hillebrand (née Tschermak) – were instructed to adjust two arrays of threads so that these arrays appeared to be parallel for the whole arrangement. Under the premise of Euclidean geometry, these alleys would have had to be de facto parallel; however, the experimental results showed systematic deviations, which implied a violation of the parallel axiom (presented in more detail in chapter 3). Schubotz (1910), Poppelreuter (1911), and Blumenfeld (1913) soon replicated Hillebrand's „alley experiments" and extended them by monocular and binocular observer conditions (cf. León, 1994). Based on these results, Luneburg (1947) developed his theory of binocular perception, which is based on the assumption of the hyperbolic Riemannian space[2] of constant curvature.

When Suppes compiled the classical studies and their answers on the topic for his review „Is visual space Euclidean?" (1977), he already failed to incorporate Hillebrand's pioneering essay. This disregard of Hillebrand's work is still being continued in pertinent textbooks on psychology of perception and cognitive neuroscience. Moreover, the phenomenon firstly described by Hillebrand is merely denoted as „Blumenfeld Alley" in contemporary encyclopedias within the discipline (e.g., Wirtz, 2017, p. 315), named after his meritorious successor Walter Blumenfeld (1882-1967). It is time to correct this. (And we may well duly write and talk about „Hillebrand-Blumenfeld-Alleys" again.)

The following section addresses the notable vita of Franz Hillebrand (Figure 1).

2 Franz Hillebrand's Biography (1863-1926)

Franz Hillebrand (born 1863 in Vienna, died in 1926 in Innsbruck) was the son of an Austrian officer. His father of the same name was a „k.k. Bergrath" (counselor of mines within the Austro-Hungarian Empire). From 1873 until 1881, young Hillebrand attended the high school in Kremsmünster in Upper Austria. In winter term 1881/82, Hillebrand began his studies at the Faculty of Philosophy at the University of Vienna, where he attended, among others, lectures of the philosophers Franz Brentano and Robert Zimmermann, as well as lectures of the classical philologists Theodor Gomperz and Wilhelm Hartel.

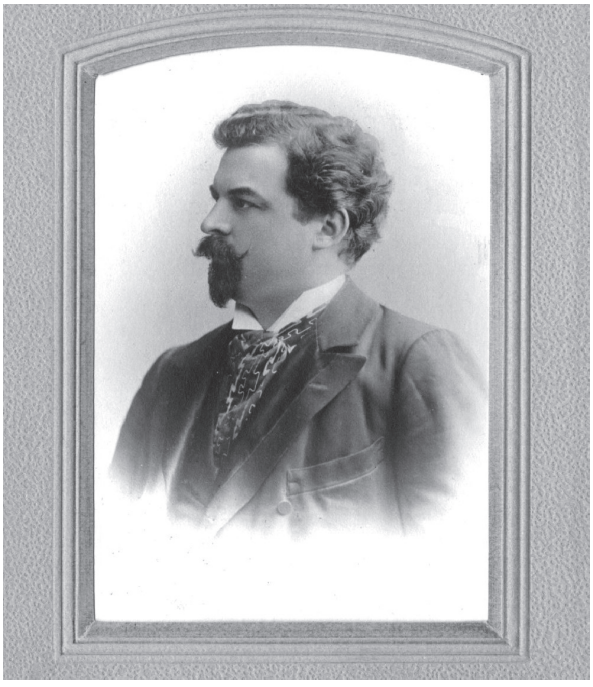

Figure 1: Franz Hillebrand (Photograph: UAI Innsbruck, without date, probably 1902).

In Vienna, Hillebrand belonged to the closer circle of students around Franz Brentano. The latter advised Hillebrand to finish his studies in Prague with Anton Marty (1847-1914), because he himself, as a Privatdozent, wasn't authorized to supervise dissertations or conduct doctoral vivas[3]. Marty, too, had been Brentano's student during his stay in Würzburg and had obtained his doctorate under the supervision of Herman Lotze (1817-1881) in 1875. In 1886, Hillebrand moved to the German University in Prague. Marty taught Hillebrand not only Brentano's philosophy, but also introduced him to experimental psychology in his modest

[2] Riemann first presented his concept during his habilitation speech on the topic „On hypothesis which underlie the geometry" (orig. „Ueber die Hypothesen, welche der Geometrie zu Grunde liegen") on June 10th, 1854, at the University of Goettingen (cf. Riemann, 1867).

[3] The former catholic priest Franz Brentano had married Ida von Lieben in 1880; because he wasn't able to legally marry by Austrian law, he was forced to reaccept Saxon citizenship and to waive his professorship that he had taken up in 1874 (cf. Oberkofler, 1986).

laboratory („cabinet“). Hillebrand obtained his doctorate of philosophy in February 1887 with his dissertation on „Synechological problems of Scholasticism“ (orig. „Synechologische Probleme der Scholastik“). His time as a postgraduate in Prague proved to be formative in so far as Hillebrand was introduced to experimental research by the physiologist Ewald Hering and the physicist Ernst Mach. In the year 1889, Hillebrand's first paper on Psychology of Perception was published: „On the specific brightness of colors – Contributions on the psychology of visual sensations“ (orig. „Über die specifische Helligkeit der Farben – Beiträge zur Psychologie der Gesichtsempfindungen“), which was highly regarded by Hering: „Since psychologists haven't applied experimental research methods for a long time yet, it is not difficult … to single out that of special value. This includes Franz Hillebrand's paper … I'm in particular prompted to this evaluation by the clear and considerate problem formulation, by which Hillebrand introduces his studies, the conscientious and sound procedure of the experiments, the unbiased and objective interpretation of the results and the logical clarity of his deductions … I believe I may express the expectation that experimental psychology will have to thank him for many advancements“ (cited by Schweinhammer, 1995). Hillebrand habilitated in 1891 at the University of Vienna with a philosophical thesis on „The new theories of categorial conclusions“ (orig. „Die neuen Theorien der kategorischen Schlüsse“), which rightfully is regarded as influenced by Brentano. Hillebrand's probationary lecture in July 1891, however, was already devoted to an experimental-psychological topic: „Adaptation as general relation between stimulus and perception“ (orig. „Die Adaption als allgemeine Beziehung zwischen Reiz und Empfindung“). Until 1894, Hillebrand was Privatdozent for philosophy at the University of Vienna. In June 1894, he was named Professor Extraordinarius of Philosophy under special consideration of experimental psychology.

On July 11, 1896, Hillebrand received a call to Tyrol under Imperial resolution: „I appoint the Professor Extraordinarius of Philosophy at the University of Vienna, Dr. Franz Hillebrand, to be Full Professor of Philosophy at the University of Innsbruck with the regular remuneration, with legal effectiveness of October 1st, 1896“ (OeStA, Allgemeines Verwaltungsarchiv, Ministerium für Cultus und Unterricht, 17550/1896). The Faculty of Philosophy of the University of Innsbruck had deliberately decided upon a philosopher with a qualification in experimental psychology. Ranked after Hillebrand were, among others, Christian von Ehrenfels (1859-1932), who had made valuable contributions with his work „On Gestalt Qualities“ (orig. „Über Gestaltqualitäten“), and Emil Arleth (1856-1909), an expert on Aristotle. Hillebrand's supporters to the milestone Innsbruck were Franz Brentano, Ewald Hering,[4] and Carl Stumpf (cf. Oberkofler, 1971). Hillebrand's inaugural lecture of October 19th, 1896, „Experimental Psychology, its origins and its tasks“ (orig. „Die experimentelle Psychologie, ihre Entstehung und ihre Aufgaben“) (UAI Nachlass Franz Hillebrand) can be seen as a key document of the period of the final disengagement of psychology from the cluster of „pure“ philosophy. Initially, Hillebrand still had devoted his regularly repeated lectures to different areas of philosophy and „general psychology“. However, after Brentano's students Emil Arleth (teaching at the University of Innsbruck from 1905 until 1909) and Alfred Kastil (in Innsbruck from 1909 until 1934) were assigned to a second philosophical professorship in Innsbruck, Hillebrand withdrew from teaching philosophy only. He perpetually announced his colloquium as a „conservatory on new phenomena in the area of psychology“ (Gatterer, Goller, & Sachse, 2018).

Per ministerial decision of February 19th, 1897, the formation of a department of experimental psychology at the University of Innsbruck was approved „in principle“; on July 9th, 1897, 1500 Gulden (old currency in Austria) were promised for the first equipment of the department (scientific appliances and books) for the following year. Furthermore, an annual endowment of 200 Gulden three times a year was guaranteed from the Ministry for Education and Teaching in Vienna. Franz Hillebrand served as initiator of the department's establishment and as first chair (Akademischer Senat der k.k. Universität Innsbruck, 1899; see also Oberkofler, 1971).

Hillebrand found a first provisional accommodation for his department in a souterrain room (Nr. 67) within the Innsbruck city hospital in the immediate vicinity of the eye clinic (Figure 2). Already a few months after his appointment in July 1896 and with support of the senate of the University of Innsbruck, Hillebrand had requested three larger rooms from the ministry which should preferably be in the proximity of the Department of Physiology, which related more to his research. Not least, he wanted to cultivate his contacts to the physiology professors at the medical faculty, for instance to Oskar Zoth (called back to Graz in 1904), Franz Bruno Hofmann (appointed to Prague in 1909), Wilhelm Trendelenburg (appointed to Gießen in 1916), or Ernst Theodor Brücke (who taught in Innsbruck from 1916 until his eviction by the NS regime in 1938). Furthermore, Hillebrand, Hoffmann und Brücke had

[4] Franz Hillebrand dedicated a great scientific epitaph with the title „Ewald Hering: Words of commemoration of the psycho physics“ (orig. „Ewald Hering: Ein Gedenkwort der Psychophysik“) (1918) for his teacher in Prague.

Figure 2: Location of the first premises of the Department of Experimental Psychology (no date) [Private property of the first author].

had the same teacher in Prague and Leipzig, Ewald Hering (UAI Sonderfaszikel Bauten, 1896).

In fall 1904, Hillebrand finally was able to move to the new premises in a new building of the Department of Physiology, Physics and Hygiene in Schoepfstraße 41 in Innsbruck (Huter, 1969). Seven years after founding the department, however, Hillebrand still had to press his demands: „The acquisition of a minimum of scientific inventories, without which such a department … cannot function, was impossible up to now, on the one hand because of too little financial resources, on the other hand, because the already far too limited room in the hospital is so extremely humid that the undersigned felt forced to temporarily place the meager equipment in a different department in order to protect it from perishing from corrosion … The laboratory is lacking a number of instruments that are constantly required; for example, instruments for time measurements and associated auxiliary devices are missing; so are element and accumulator units, inductors, switching and contact apparatuses etc. Furthermore, certain additions in the library, some of them rather costly, are entirely indispensable: for instance, the new acquisition of the first 18 volumes of the Journal „Zeitschrift für Psychologie und Physiologie der Sinnesorgane und der Wundt'schen ‚Studien'" (21 volumes[5])" (as cited in Schweinhammer, 1995, p. 99). Hillebrand's difficult situation becomes also apparent in a letter to Alexius Meinong in the year 1904: „Working fails because of pecuniary details, such as that I'm not granted an attendant or an equivalent aide. I'm tired of struggling with such tribulations and thus issued an ultimatum" (UBG, Nachlass Alexius Meinong, XLI, Nr. 1855). Furthermore, Hillebrand had to travel to the library of the University of Vienna in order to prepare his experimental studies and publications because the local subject-specific book collection was still highly inadequate. Only with the approval of an extraordinary endowment for the years 1906 until 1908, the situation at the Department of Psychology somewhat eased (Schweinhammer, 1995).

The briefly described initial phase of the department required a lot of effort and time resources of the department chair. Hillebrand also personally paid a high price for the initially undesirable work place: „He contracted a severe joint disease in the humid and unheated room" (Oberkofler, 1971, p. 167).

In April 1910, the 4th congress for experimental psychology, organized by Hillebrand, took place in Innsbruck. The program included thirty-three highly qualified talks, a scientific discussion by prominent congress participants, and an exhibition of scientific apparatuses (cf. Schumann, 1911).[6] Hillebrand's research in Innsbruck and his resulting publications focused on the experimental investigation of space perception; his basic studies on psychology of perception were widely acknowledged in the scientific community.

Hillebrand would have been able to produce even more comprehensive research and publications, if he had had a well-furnished laboratory, just like the excellently equipped one of the University of Munich. Thus, it was not by chance that he was interested in a vacant chair in Munich in 1910, for which, however, Oswald Külpe was eventually appointed in 1913. In

5 Wundt, W. (1881-1902). Philosophische Studien. Leipzig: Engelmann.

6 Fifty-three out of the 128 congress participants in Innsbruck were member of the „Gesellschaft für experimentelle Psychologie" (Society of Experimental Psychologists), which had been founded in 1904 in Gießen under the aegis of Georg Elias Müller and which is now known as the „Deutsche Gesellschaft für Psychologie" (German society of psychology).

particular Franz Brentano and Ernst Mach had supported Hillebrand's plan.

In 1913, Hillebrand published a pamphlet against the „lockout of psychologists". It was the only publication during his time in Innsbruck that dealt with philosophical questions. With this paper, Hillebrand responded sharply to the „Declaration of the hundred and six" (philosophers), who, led by the philosophy professors Rudolf Eucken, Edmund Husserl, Paul Natorp, Heinrich Rickert, Alois Riehl, and Wilhelm Windelband, demanded to keep representatives of experimental psychology away from philosophical professorships (UAI, Nachlass Franz Hillebrand). This was an attempt to deprive experimental psychology of any scientific organizational support. In the same year, 1913, Wilhelm Wundt had already pointedly summarized the controversy in his essay „Psychology in its struggle of existence" (orig. „Die Psychologie im Kampf ums Dasein"): „The philosophers apparently see themselves threatened in their status quo" (p. 2), „Conducting experiments is a philistine art; thus, an experimental psychologist is a scientific craftsman at most. But a craftsman doesn't fit among the philosophers" (p. 9). Wundt and Hillebrand mutually argued in favor of the preservation of the status quo, the education of prospective psychologists within the scope of philosophy.

In 1922 (a, b), Franz Hillebrand, already in poor health, published the essay „On the theory of stroboscopic movements" (orig. „Zur Theorie der stroboskopischen Bewegungen"). With this publication, he stepped directly into the dispute about Max Wertheimer's „Experimental studies on the seeing of movement" (orig. „Experimentelle Studien über das Sehen von Bewegung") (1912), which is a discussion on the so-called phi phenomenon (apparent motion). The essential part of this phenomenon, which had been discovered by Sigmund Exner in 1875, is that two originally separate stimuli are seen as one stimulus, that is, a *phenomenal identity* is achieved. While Hillebrand interpreted the phi phenomenon on the basis if the „theory of wandering attention", Wertheimer explained it by means of the „Short-circuit theory". Both concepts were experimentally disconfirmed by Theodor Erismann in 1948 in Innsbruck. The fact that the distance of the stroboscopic apparent motions is not always the shortest can be seen as an argument against the short-circuit theory. It was argued against the theory of wandering attention, that *opposed* stroboscobic motions can be seen *at the same time* within the same field of attention. On basis of the analyses of the phi phenomenon, Wertheimer developed the basic ideas of the Gestalt theory. His achievement was to see motion as a distinct, irreducible phenomenon. It was self-evident that the Gestalt psychologist Wolfgang Köhler intervened in the dispute. He addressed a letter to Hillebrand in October 1922: „It takes quite some effort … to understand this sophisticated deduction of the stroboscopic effect. I will see to it that I manage even better yet, because I would like to present your work in the 'Psychol. Forschung' (Psychological Research)." At the same time, he asked Hillebrand to clarify several issues in order to „overcome these concerns". In February 1923, Köhler informed him that Wertheimer himself would be writing the response to Hillebrand's essay. „Without any doubt, your views will give cause to lively discussions, …" Kurt Koffka from Gießen, a further co-founder of the Gestalt psychology whom Hillebrand had critically reviewed in his work, reacted promptly as well and expressed his hope that this work would „bring us nearer to a theoretical decision". In this concern he felt misunderstood by Hillebrand (UAI, Nachlass Franz Hillebrand).

Several of Hillebrand's students in Innsbruck chose an academic career path: Hans Rupp [1880-1954, later assistant with Carl Stumpf in Berlin and as Professor Extraordinarius in Berlin editor of the Psychotechnische Zeitschrift (Psycho Technical Journal)], Richard Strohal (1888-1976, since 1930 professor of philosophy with particular consideration of pedagogics in Innsbruck), and Konstantin Radakovic (1894-1973, later professor of philosophy in Graz).

Hillebrand's second marriage in December 1920 was to his former student and later associate, Franziska Hillebrand, née von Reicher. In 1919, in the age of 34, she had received her Ph.D. with a dissertation supervised by Alfred Kastil on „Turning away from the non-real" (orig. „Die Abkehr vom Nichtrealen") – a critique of Edmund Husserl and Alexius Meinong that followed Brentano.

After her husband's death on April 13th, 1926, Franziska Mayer-Hillebrand redacted and published his late experimental work as „Doctrine of visual sensation" (orig. „Lehre von den Gesichtsempfindungen") (1929) based on the records he had left behind. „With Franz Hillebrand, experimental psychology has lost one of its most considerate and methodologically conscientious researchers. Building its foundation a long time ago on not so solid ground, he created several exemplary pieces of work with the little means he had available … as a straight forward man, down to earth, reluctant towards compromises, but also personally not the most sociable person, he went a quite lonely way", his friends and colleagues of many years, Carl Stumpf and Hans Rupp, wrote in their obituary in the Zeitschrift für Psychologie (Journal of Psychology) (1927, p. 1), at which Hillebrand had played an active part for three decades (on the biographical note Hillebrand see UAI, Nachlass Franz Hillebrand; UAI, Goldenes Buch der Universität Innsbruck (from 1775); UAW, Philosophisches Nationale, Franz Hillebrand, 1881/1882; Goller, 1989; Gatterer, Goller & Sachse, 2018).

It was Theodor Erismann (1883-1961) who succeeded the professorship in Innsbruck in 1927 (cf. Sachse, Beermann, Martini, Maran, Domeier & Furtner, 2017).

3 The alley experiments

A preliminary note: It is generally assumed that the visual spatial perception of objects corresponds well with the underlying physical conditions. Several perception phenomena prove, however, that this is not always the case [just think of the „moon illusion" (Ross & Plug, 2002) or the „Ames room" (Goldstein, 2010)]. The correlation between the stimulus conditions in physical space and the visual spatial perception of these stimulus conditions (i.e., the visual field) can be described by psychophysical relations. These project the structure of the physical space into the structure of the perceived space. The structure of spatial relations of objects in physical space in those sections relevant for perception can be well described by Euclidian geometry. In contrast, the structure of the visual space that corresponds to this physical space is not yet determined with certainty. If the investigation of such a structure of perception is possible, it affects the geometry of visual space (Lukas, 1983, 1996, 2001; Zimmer, 1998).

Hillebrand's credo, „the conditions of everyday life are always complicated and it is only the art of the experimenter that can simplify them" (UAI Nachlass Franz Hillebrand, Inaugural address from October 19th, 1896) without holds true also for his „alley experiments". In order to investigate the relations between physical space and the visual space, Hillebrand, in 1900, proceeded as follows: Under standardized conditions, he presented two arrays of threads as an alley (with an exactly symmetrical distance from the principal line of sight; eyes in primary position). The observation happens in a way that any surrounding influences other than those threads were eliminated. Hillebrand was able to prove that under these conditions, participants by no means perceived a regular, i.e., a symmetrical and orthogonal alley. In order to determine the deviation, the participants had to direct the spacings between the threads in a way so that phenomenal equidistance was achieved. This revealed that the spacings grew with increasing distance, and in particular that the *lateral* spacings diverged more and more (see Figure 3). More precisely: Whenever the threads are supposed to margin an orthogonal rectangle, then they have to lie towards the frontal line of sight in the shape of a convex arc, whose curvature declines with increasing spacing (Hillebrand, 1902; see also Blumenfeld, 1913; Köllner, 1923; Hofmann, 1925; Dittler, 1932; Klix, 1962).

Criticism about this „interesting, but not entirely easily readable work" (S. 367) came from the physiologist Johannes von Kries (1903). In detail, he not only challenged the implied constancy assumption, but also the equation of the significant differences in depth. In his opinion, the view that the same sizes of lateral disparity correspond to the same depth of vision is incorrect; on the contrary, the results of the lateral disparity are different for different depths of vision. Blumenfeld's study (1913), however, proved that Hillebrand's theory remains valid even when equal and just noticeable depth differences don't correspond the same parallel axis.

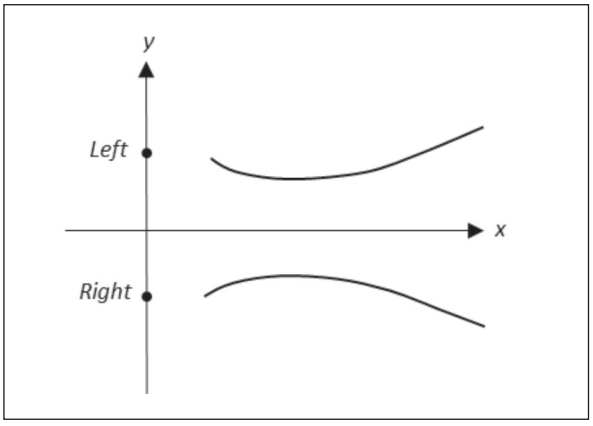

Figure 3: Hillebrand's alley curves (adapted from Luneburg, 1948, p. 217).

Hillebrand repeated his experiments by using little gas flames in the dark instead of the black threads so that criteria of prior experiences could be excluded to an even greater extent. This subsequent experiment, which Blumenfeld (1913) relied on in terms of content as well as concerning the utilized apparatus[7], unfortunately wasn't published by Hillebrand. In this experiment, by moving little lamps, the participants created a stimulus configuration which – depending on the instruction – were labelled either *parallel alleys* or *distance alleys*. (These specific terms, however, were only introduced by Blumenfeld, 1913). Here, two light points P and P' with the coordinates (were presented in a horizontal plane at eye level of the participants. The y-axis is directed *straight ahead* and the participant is located in the origin of the coordinate system (see Figure 4, upper panel). In the *parallel alley*, partici-

[7] Blumenfeld (1913, p. 275): „For this purpose, an apparatus that was already available at the Berlin department of psychology seemed suitable, which had been built based on a specimen designed and used by Hillebrand in Innsbruck and which differs from the latter only by marginal technical improvements partly already suggested by Hillebrand himself because of his experiences with the Innsbruck apparatus."

pants arranged all further light points in such a manner that they perceived them as two straight lines parallel to the y-axis. The newly arranged points stayed visible for the rest of the experimental trial, so that participants always saw all arranged points and were able to apply corrections afterwards. Likewise, in the *distance alley* condition, two points were presented symmetrical to the y-axis. Further pairs of points P_i, P_i' were arranged by the participant, so that the distance between P_i and P_i' appeared the same as the distance between P and P'. However, the already arranged points were not presented to the participant anymore, so that he or she only saw two pairs of points at a time: the fixed points P, P', and the pair of points to be arranged at a given time, P_i, P_i' (see Lukas, 1996, 2001). The difference between the parallel alley and the distance alley is depicted in Figure 4 (lower panel).

In other words: Per definition, the two sequences of points of a parallel alley appeared to the participants as straight lines that neither converged nor diverged. When asked for the *lateral* spacings of the points to each other, however, participants reported that the spacing was larger for the points the furthest away than for the points most at the front. This phenomenon turned out to be extremely stable and robust against variations of the experimental design; it was replicated in subsequent studies and extended by further conditions of observation (among others, by Hardy, Rand & Rittler, 1951; Hardy, Rand, Rittler, Blank & Boeder, 1953; Squires, 1956; Zajaczkowska, 1956 a, b; Shipley, 1957; Indow, Inoue & Matsushima, 1962, 1963; Kienle, 1968; Battro, di Pierro Netto & Rozenstraten, 1976; Indow, 1979, 1988; Indow & Watanabe, 1984 a, b; Higashiyama, Ishikawa & Tanaka, 1990; Cuijpers, Kappers & Koenderink, 2000, 2001, 2002; Koenderink, van Doorn, de Ridder & Oomes, 2010; Erkelens, 2015 a). The results of the alley experiments indicate that the geometry of visual space can't be Euclidian.

For the mathematician Luneburg (1947, 1948, 1950), these results were *the* piece of evidence for a hyperbolic structure of space perception, because in elliptic geometry, distance alleys are located *within* parallel alleys, while in Euclidean geometry, both alleys would have to correspond. He concluded: the existence of a Riemannian space with constant curvature. It remains Luneburg's achievement to have formulated the determining coherence between the perception experiments as well as the possible geometry of perception. Luneburg's theory of binocular perception was completed, modified and advanced by Blank (1953, 1957, 1958 a / b, 1959, 1961, 1978). Furthermore, the relevant experimental studies and conceptional considerations on spatial relations by Foley (1964, 1966, 1972, 1980, 1991) and Foley, Ribeiro-Filho and Silva (2004) are rooted in this tradition; these, too, suggest the assumption of a perception space that is

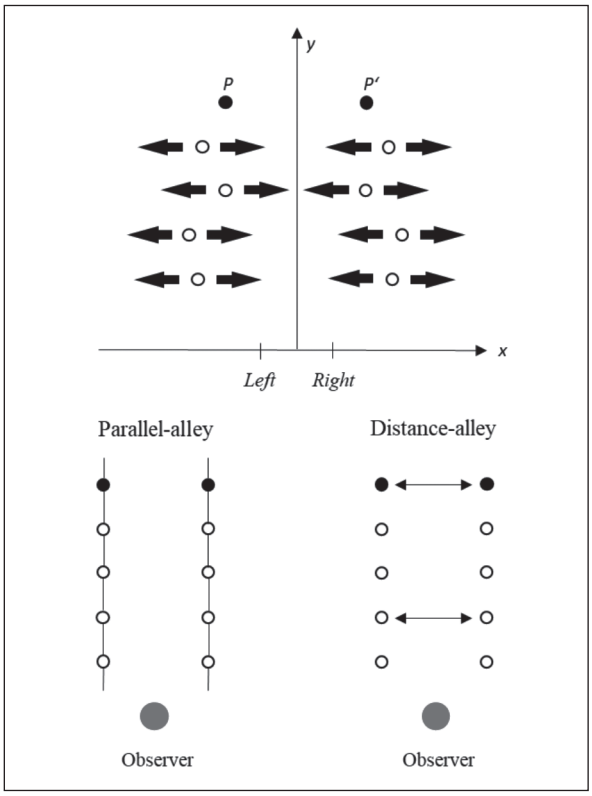

Figure 4: Simplified experimental set-up for the alley experiments (adopted from Lukas, 2001, p. 198).

inhomogeneous with regard to the geometry, and that is supposedly Euclidean or hyperbolic.

In a current, groundbreaking study, yet again based on the classical alley experiments, Erkelens (2015 a) described *perspective space* as a viable model of visual space. The alleys that he calculated within this framework mostly corresponded with the experimentally generated alleys. The geometry of perspective space, however, differs substantially from the geometry of the Euclidean space.

Currently, within scientific literature, geometric models of visual perception are mostly discussed in the context of mathematical and of cognitive psychology, occasionally also in neuroscience, but rarely in relation to philosophical, epistemological questions.

4 Conclusion

From the year 1900, the experimental psychologist Franz Hillebrand conducted studies at the University of Innsbruck in which he was the first to systematically analyze the problem of the non-Euclidean (hyperbolic) space perception by means of „alley experiments". Euclid had already postulated a perception theory in his „Optic" which were according to the axioms of his geometry (see in detail Gray, 2004; Greenberg, 2008, 2010). However, when the visual space perception

doesn't correspond to the axioms (in particular the parallel axioms) of the Euclidean geometry, then this is called non-Euclidean space perception. It was Helmholtz in the second half of the 19th century who took up Riemann's hypothesis on the foundation of geometry and found that physical (and thus perceived) spaces require the „free mobility" of solid bodies. Thus, the possible geometries are limited to the Euclidean, spherical, and hyperbolic geometries (Zimmer, 2017).

Experiments on perception psychology conducted by Hillebrand in Innsbruck with the utmost precision (and later, among others, by Blumenfeld in Berlin) revealed that parallel lines in Euclidian space appeared curved; this result demonstrates a non-Euclidean structure. The fact that equidistantly arranged spacings are objectively broader than in a configuration with parallel lines proves the negative curvature, or more specifically, the hyperbolic structure of the phenomenal space.

The theory of a non-Euclidean visual space did not remain unchallenged, but is nonetheless supported by a variety of current experimental studies. Critics emphasize the challenge to characterize the visual space globally with a particular geometry, but also to point out the problem of the dependency of visual space on context (see for example Suppes, 1995; Lukas, 1996, 2001; Erkelens, 2015 a). Wagner (2006) insists: „Our goal should be to find not *the* geometry of visual space, but the *geometries* of visual space" (p. 230). In this sense, the Innsbruck „alley experiments" almost 120 years ago only provided the impetus to the question of the geometrical structure of visual space which still has not been clarified in detail so far.

By Hillebrand's „alley experiments", research on space perception, on geometry of visual-spatial orientation, on size constancy and on model development in mathematics and development of theories, such as in the area of perception psychology, received internationally recognized and important impulses (see as a summary Indow, 1991, 2004; Wagner, 2006).

For some time, research on visual perception has not been the object of a sole scientific discipline. Insofar, interdisciplinary approaches for further research questions on visual space perception might prove worthwhile, which – possibly not in a reductionist way – would take greater account of several areas of research, such as a) the neuronal basics of visual perception (cf. Parr & Friston, 2017), b) visual information intake and processing (cf. Zimmermann & Lappe, 2016; Neilson, Neilson & Bye, 2018), c) the role of processes of perception per se, d) the relation of perception and action, e) in more detail, the actual role of the perspective space as a model of the visual space (Erkelens, 2015 a, b, c), and f) possible aspects of application.

Acknowledgements

We would like to thank Dr. Matthias Mösch (University of Innsbruck, Department of English) cordially for his valuable input in terms of language.

References

Akademischer Senat der k.k. Universität Innsbruck (1899). *Die Leopold-Franzens-Universität zu Innsbruck in den Jahren 1848-1898; Festschrift aus Anlass des 50jährigen Regierungs-Jubiläums Sr. Majestät des Kaisers Franz Joseph I.* Innsbruck: Wagner.

Battro, A. M., Netto, S. di P. & Rozestraten, R. J. A. (1976). Riemannian geometries of variable curvature in visual space: Visual alleys, horopters, and triangles in big open fields. *Perception*, *5*, 9-23.

Blank, A. A. (1953). The Luneburg theory of binocular visual space. *Journal of the Optical Society of America*, *43*, 717-727.

Blank, A. A. (1957). A geometry of vision. *The British journal of physiological optics*, *14*, 222-235.

Blank, A. A. (1958 a). Axiomatics of binocular vision: The foundations of metric geometry in relation to space perception. *Journal of the Optical Society of America*, *48*, 328-334.

Blank, A. A. (1958 b). Analysis of experiments in binocular space perception. *Journal of the Optical Society of America*, *48,* 911-925.

Blank, A. A. (1959). Luneburg theory of binocular space perception. In S. Koch (Ed.), *Psychology: A study of a science* (pp. 395-426). New York: McGraw-Hill.

Blank, A. A. (1961). Curvature of binocular visual space: An experiment. *Journal of the Optical Society of America*, *51*, 335-339.

Blank, A. A. (1978). Metric geometry in human binocular perception: Theory and fact. In E. L. J. Leeuwenberg & H. F. J. M. Buffart (Eds.), *Formal theories of visual perception* (pp. 83-102). New York: Wiley.

Blumenfeld, W. (1913). Untersuchungen über die scheinbare Größe im Sehraume. *Zeitschrift für Psychologie*, *65*, 241-404.

Cuijpers, R. H., Kappers, A. M. L. & Koenderink, J. J. (2000). Large systematic deviations in visual parallelism. *Perception*, *29*, 1467-1482.

Cuijpers, R. H., Kappers, A. M. L. & Koenderink, J. J. (2001). On the role of external reference frames on visual judgments of parallelity. *Acta Psychologica*, *108*, 283-302.

Cuijpers, R. H., Kappers, A. M. L. & Koenderink, J. J. (2002). Visual perception of collinearity. *Perception & Psychophysics*, *64*, 392-404.

Dittler, R. (1932). Die Physiologie des optischen Raumsinnes. In F. Schieck (Hrsg.), *Kurzes Handbuch der Ophthalmologie* (Bd. 2), Physiologie; Optik; Untersuchungsmethoden; Bakteriologie (S. 378-459). Berlin: Springer.

Erismann, T. (1948). Die Stroboskopie und ihre Erklärung aus einer Wahrnehmungstheorie. In *Kongressbericht; Berufsverband Deutscher Psychologen, Bonn 29. August bis 2. September 1947* (S. 32-49). Hamburg: Nölke.

Erkelens, C. J. (2015 a). The perspective structure of visual space. *i-Perception*, *6*, 1-13.

Erkelens, C. J. (2015 b). The extent of visual space inferred from perspective angles. *i-Perception*, *6*, 5-14.

Erkelens, C. J. (2015 c). Perception of perspective angles. *i-Perception*, *6*, 1-11.

Foley, J. M. (1964). Desarguesian property in visual space. *Journal of the Optical Society of America*, *54*, 684-692.

Foley, J. M. (1966). Locus of perceived equidistance as a function of viewing distance. *Journal of the Optical Society of America*, *56*, 822-827.

Foley, J. M. (1972). The size-distance relation and intrinsic geometry of visual space: Implications for processing. *Vision Research*, *12*, 323-332.

Foley, J. M. (1980). Binocular distance perception. *Psychological Review, 87*, 411-434.

Foley, J. M. (1991). Binocular space perception. In D. Regan & J. R. Cronly-Dillon (Eds.), *Binocular vision: Vision and visual dysfunction,* Vol. 9 (pp. 75-92). Basingstoke: Macmillan.

Foley, J. M., Ribeiro-Filho, N. P. & Silva, J. A. (2004). Visual perception of extent and the geometry of visual space. *Vision Research*, *44*, 147-156.

Gatterer, J., Goller, P. & Sachse, P. (2018). Franz Hillebrand: die experimentelle Psychologie, ihre Entstehung und ihre Aufgaben. Antrittsvorlesung, gehalten am 19. Oktober 1896 in Innsbruck. *Journal Psychologie des Alltagshandelns*, *11* (1), 47-63.

Goldstein, E. B. (2010). *Sensation and perception* (8. ed.). Belmont: Wadsworth Cengage Learning.

Goller, P. (1989). *Die Lehrkanzeln für Philosophie an der Philosophischen Fakultät der Universität Innsbruck (1848 bis 1945)*, Forschungen zur Innsbrucker Universitätsgeschichte 15. Universität Innsbruck.

Gray, J. J. (2004). *János Bolyai, non-Euclidean geometry, and the nature of space*. Cambridge: Burndy Library.

Greenberg, M. J. (2008). *Euclidean and non-Euclidean geometries; development and history*. New York: Freeman.

Greenberg, M. J. (2010). Old and new results in the foundations of elementary plane Euclidean and Non-Euclidean geometries. *The American Mathematical Monthly*, *117*, 198-219.

Hardy, L. H., Rand, G. & Rittler, M. C. (1951). Investigation of visual space: The Blumenfeld alley. *Archives of Ophthalmology*, *45*, *53-63*.

Hardy, L. H., Rand, G., Rittler, M. C., Blank, A. A. & Boeder, P. (1953). *The geometry of binocular space perception*. New York: Institute of Ophthalmology, Columbia University.

Helmholtz, H. von (1884). Über den Ursprung und die Bedeutung der geometrischen Axiome. In *Vorträge und Reden*, Band II, 3. Auflage (S. 1-31). Braunschweig: Vieweg.

Higashiyama, A., Ishikawa, T. & Tanaka, K. (1990). Visual alleys as a function of instructions under informative and reduced conditions of viewing. *Perception & Psychophysics*, *47*, 468-476.

Hillebrand, F. (1889). *Über die specifische Helligkeit der Farben – Beiträge zur Psychologie der Gesichtsempfindungen* (S. 1-51*).* Sitzungsberichte der kaiserlichen Akademie der Wissenschaften in Wien, Mathematisch- naturwissenschaftliche Classe, Abteilung 3, Band XCVIII. Wien: K. K. Hof- und Staatsdruckerei.

Hillebrand, F. (1891). *Die neuen Theorien der kategorischen Schlüsse.* Wien: Alfred Hölder, k.u.k. Hof- und Universitäts-Buchhändler.

Hillebrand, F. (1894). *Das* Verhältnis von Accommodation und Konvergenz zur Tiefenlokalisation. *Zeitschrift für Psychologie und Physiologie der Sinnesorgane*, *7*, 97-151.

Hillebrand, F. (1902). *Theorie der scheinbaren Grösse bei binocularem Sehen.* Denkschriften der Kaiserlichen Akademie der Wissenschaften, Mathematisch-naturwissenschaftliche Klasse, 72 (S. 255-307). Wien: Kaiserlich-Königliche Hof- und Staatsdruckerei.

Hillebrand, F. (1913). Die Aussperrung der Psychologen. *Zeitschrift für Psychologie*, *67*, 1-21.

Hillebrand, F. (1918). *Ewald Hering: Ein Gedenkwort der Psychophysik.* Berlin: Springer.

Hillebrand, F. (1922 a). Zur Theorie der stroboskopischen Bewegungen. *Zeitschrift für Psychologie*, *89*, 209-272.

Hillebrand, F. (1922 b). Zur Theorie der stroboskopischen Bewegungen. *Zeitschrift für Psychologie*, *90*, 1-66.

Hillebrand, F. (1929). *Lehre von den Gesichtsempfindungen.* Auf Grund hinterlassener Aufzeichnungen von Franz Hillebrand, herausgegeben von Franziska Hillebrand. Wien: Springer.

Hofmann, F. B. (1925). *Die Lehre vom Raumsinn des Auges*, Teil 2. Berlin: Springer.

Holtz, W. (1893). Ueber den unmittelbaren Größeneindruck in seiner Beziehung zur Entfernung und zum Contrast. *Nachrichten von der Königl. Gesellschaft der Wissenschaften und der Georg-Augusts-Universität zu Göttingen* (S. 159-167).

Huter, F. (1969). *Hundert Jahre Medizinische Fakultät Innsbruck, 1869 bis 1969*, Teil 1 (Forschungen zur Innsbrucker Universitätsgeschichte 7). Innsbruck: Österreichische Kommissionsbuchhandlung.

Indow, T. (1979). Alleys in visual space. *Journal of Mathematical Psychology*, *19*, 221-258.

Indow T. (1988). Alleys on apparent frontoparallel plane. *Journal of Mathematical Psychology*, *32*, 259-284.

Indow, T. (1991). A critical review of Luneburg's model with regard to global structure of visual space. *Psychological Review*, *98* (3), 430-453.

Indow, T. (2004). *The global structure of visual space*. River Edge: World Scientific.

Indow, T., Inoue, E. & Matsushima, K. (1962). An experimental study of the Luneburg theory of binocular space perception: The 3- and 4-point and the alley experiments. *Japanese Psychological Research*, *4*, 6-24.

Indow, T., Inoue, E. & Matsushima, K. (1963). An experimental study of the Luneburg theory of binocular space perception (3). The experiments in a spacious field. *Japanese Psychological Research*, *5*, 10-27.

Indow, T. & Watanabe, T. (1984 a). Parallel- and distance-alleys with moving points in the horizontal plane. *Perception & Psychophysics*, *35*, 144-154.

Indow, T. & Watanabe, T. (1984 b). Parallel- and distance-alleys on horopter plane in the dark. *Perception*, *13*, 165-182.

Kienle, G. (1968). *Die optischen Wahrnehmungsstörungen und die nichteuklidische Struktur des Sehraumes*. Stuttgart: Thieme.

Klix, F. (1962). *Elementaranalysen zur Psychophysik der Raumwahrnehmung*. Berlin: Deutscher Verlag der Wissenschaften.

Köllner, H. (1923). Über die Lage scheinbar paralleler nach der Tiefe verlaufender Linien und ihre Beziehung zu den Sehrichtungen. *Pflügers Archiv für die gesamte Physiologie des Menschen und der Tiere*, CXGVII, 518-535.

Koenderink, J., van Doorn, A., de Ridder, H. & Oomes, S. (2010). Visual rays are parallel. *Perception*, *39*, 1163-1171.

Kries, J. von (1891). Beiträge zur Lehre vom Augenmass. *Beyträge zur Psychologie und Physiologie der Sinnesorgane*, 173-193.

Kries, J. von (1903). Literaturbericht: F. Hillebrand. Theorie der scheinbaren Größe bei binokularem Sehen. Denkschriften der mathematisch-naturwissenschaftlichen Klasse der Wiener Akademie, 72, 1902. *Zeitschrift für Psychologie und Physiologie der Sinnesorgane*, 33, 366-368.

León, R. (1994). Zwei Themen des Briefwechsels zwischen Walter Blumenfeld und Franziska Baumgarten-Tramer. In H. Gundlach (Hrsg.), *Arbeiten zur Psychologiegeschichte* (S. 23-38). Göttingen: Hogrefe-Verlag.

Lukas, J. (1983). Visuelle Frontalparallelen: Ein Entscheidungsexperiment zu den Theorien von Blank, Foley und Luneburg. *Zeitschrift für Experimentelle und Angewandte Psychologie*, *30*, 610-627.

Lukas, J. (1996). *Psychophysik der Raumwahrnehmung*. Weinheim: Beltz.

Lukas, J. (2001). Geometry of visual space. In *International Encyclopedia of the Social & Behavioral Sciences*, Vol. 24 (pp.197-200). Amsterdam: Elsevier.

Luneburg, R. K. (1947). *Mathematical analysis of binocular vision*. Princeton: Princeton University Press.

Luneburg, R. K. (1948). Metric methods in binocular visual perception. In *Studies and Essays, Courant Anniversary Volume* (pp. 215-240) New York: Interscience.

Luneburg, R. K. (1950). The metric of binocular visual space. *Journal of the Optical Society of America*, *40*, 627-642.

Martius, G. (1889). Ueber die scheinbare Grösse der Gegenstände und ihre Beziehung zur Grösse der Netzhautbilder. *Philosophische Studien*, *5*, 601-617.

Neilson, P. D., Neilson, M. D. & Bye, R. T. (2018). A Riemannian geometry theory of three-dimensional binocular visual perception. *Vision*, *2*, 43.

Oberkofler, G. (1971). Franz Hillebrand (1863-1926). Der Begründer des Instituts für Experimentelle Psychologie in Innsbruck. In F. Huter (Hrsg.), *Die Fächer Mathematik, Physik und Chemie an der Philosophischen Fakultät zu Innsbruck bis 1945*, S. 163-171 (Forschungen zur Innsbrucker Universitätsgeschichte 10). Innsbruck: Österreichische Kommissionsbuchhandlung.

Oberkofler, G. (1986). Aus Briefen von Ewald Hering an Franz Hillebrand. In G. Hamann (Hrsg.), *Aufsätze zur Geschichte der Naturwissenschaften und Geographie* (S. 184-203). Wien: Verlag der Österreichischen Akademie der Wissenschaften.

Parr, T. & Friston, K. J. (2017). The active construction of the visual world. *Neuropsychologia*, *104*, 92-101.

Poppelreuter, W. (1911). Beiträge zur Raumpsychologie. *Zeitschrift für Psychologie*, 58, 200-262.

Riemann, B. (1867). *Ueber die Hypothesen, welche der Geometrie zu Grunde liegen.* Göttingen: Dietrich.

Ross, H. & Plug, C. (2002). *The mystery of the moon illusion; exploring size perception*. Oxford: Oxford University Press.

Sachse, P., Beermann, U., Martini, M., Maran, T., Domeier, M. & Furtner M. R. (2017). „The world is upside down" – The Innsbruck Goggle Experiments of Theodor Erismann (1883-1961) and Ivo Kohler (1915-1985). *Cortex*, *92*, 222-232.

Schubotz, F. (1910). Beiträge zur Kenntnis des Sehraumes auf Grund der Erfahrung. *Archiv für die gesamte Psychologie*, *22*, 101-149.

Schumann, F. (1911). *Bericht über den IV. Kongreß für Experimentelle Psychologie in Innsbruck vom 19. bis 22. April 1910.* Leipzig: Barth.

Schweinhammer, S. (1995). *Die Geschichte des Instituts für Experimentelle Psychologie an der Universität Innsbruck.* Die Anfangsjahre: 1897 bis 1926. Diplomarbeit, Universität Wien.

Shipley, T. (1957). Convergence function in binocular visual space: II. Experimental report. *Journal of the Optical Society of America*, *47*, 804-821.

Squires, P. C. (1956). Luneburg theory of visual geodesics in binocular space perception. *Archives of Ophthalmology*, *56*, 288-297.

Stumpf, C. & Rupp, H. (1927). Franz Hillebrand †. *Zeitschrift für Psychologie*, *102*, 1-5.

Suppes, P. (1977). Is visual space Euclidean? *Synthese*, *35* (4), 397-421.

Suppes, P. (1995). Some foundational problems in the theory of visual space. In R. D. Luce, M. D'Zmura, D. Hoffman, G. J. Iverson & A. K. Romney (Eds.), *Geometric representations of perceptual phenomena: Papers in honor of Tarow Indow on his 70th birthday* (pp. 37-45). Mahwah: Erlbaum.

Wagner, M. (2006). *The geometries of visual space*. Mahwah: Erlbaum.

Wertheimer, M. (1912). Experimentelle Studien über das Sehen von Bewegung. *Zeitschrift für Psychologie*, *61*, 161-265.

Wirtz, M. A. (2017). *Dorsch – Lexikon der Psychologie* (18., überarb. Aufl.). Bern: Hogrefe.

Wundt, W. (1881-1902). Philosophische Studien. Leipzig: Engelmann.

Wundt, W. (1913). *Die Psychologie im Kampf ums Dasein.* Leipzig: Kröner.

Zajaczkowska, A. (1956 a). Experimental determination of Luneburg's constants σ and κ. *Quarterly Journal of Experimental Psychology*, *8* (2), 66-78.

Zajaczkowska, A. (1956 b). Experimental test of Luneburg's theory. Horopter and alley experiments. *Journal of the Optical Society of America*, *46*, 514-527.

Zimmer, A. (2017). Raumwahrnehmung, nichteuklidische. In M. A. Wirtz (Hrsg.), *Dorsch – Lexikon der Psychologie* (18., überarb. Aufl., S.1407f). Bern: Hogrefe.

Zimmer, K. (1998). *Experimentelle Untersuchungen zur geometrischen Struktur des binokularen Sehraums*. Aachen: Shaker.

Zimmermann, E. & Lappe, M. (2016). Visual space constructed by saccade motor maps. *Frontiers in Human Neuroscience*, *10*, 225.

Archival Materials

OeStA Österreichisches Staatsarchiv
- Allgemeines Verwaltungsarchiv, Ministerium für Cultus und Unterricht, 17550/1896.

UAI – Universitätsarchiv Innsbruck
- Nachlass Franz Hillebrand.
- Sonderfaszikel Bauten, 16. Dezember 1896.
- Goldenes Buch der Universität Innsbruck (ab 1775).

UAW – Universitätsarchiv Wien
- Nationale (Franz Hillebrand, Wintersemester 1881/1882, Philosophische Fakultät).

UBG – Universitätsbibliothek Graz
- Nachlass Alexius Meinong: XLI, Nr. 1855 Brief Hillebrands an Meinong (3. Dezember 1904).

Correspondence to:
Univ.-Prof. Dr. Pierre Sachse
Leopold-Franzens-University Innsbruck
Department of Psychology
Innrain 52 f
A-6020 Innsbruck
Pierre.Sachse@uibk.ac.at